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LETTER TO THE EDITOR

Paper crushes fractally

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Abstract. The geometry of crumpled paper balls is investigated. It is shown that these systems are fractals and their properties are studied.

Fractals are now a topic of wide interest (Mandelbrot 1983) and here we describe an interesting new example of fractal dimension defined via the mass-size exponent. This example is based on 4361 measurements of mass (M) and diameter ($2R$) from 89 experiments with crumpled paper balls. From every experiment a log-log graph of diameter against mass is generated, using seven coordinate pairs ($2R_i$, M_i), $i = 1, 2, \ldots, 7$; $M_i = 1, 2, 4, \ldots, 64$. Each of these diameters is in turn an averaged value obtained from measurements along seven different directions taken at random. In each of these experiments we find a simple relationship between diameter and mass of the form $2R_i = k_j M_i^{1/D_j}$, with $j = 1, 2, \ldots, 89$. D_j is interpreted as the fractal dimension of the balls in the jth experiment. From these 89 values of D_j and k_j we obtain the average values for D and k, with the corresponding mean square deviations ($D = 2.51 \pm 0.19$, $k = 5.75 \pm 0.71$) for normal writing paper of surface density $\sigma \simeq 80\ \mathrm{g\ m^{-2}}$. The fractal dimension D in this case tells us about the complexity or degree of contortion of the area, since a fixed measure of rounded smooth area can enclose a larger volume than a complicated one can. It is experimentally evident that k has a percentage mean square deviation (lacunarity) approximately two times larger than that of $D(\Delta k/k) = 0.12$ while $(\Delta D/D) = 0.07$. These values show that D is much less affected by the way of crumpling (pressure applied, haste or not, etc) than k is. The topological dimension of these balls is $d_T = 2$, since they are made of sheets of paper, which conform with $d_T = 2$. On the other hand, they are embedded in the Euclidean tridimensional space ($d = 3$), so their fractal D satisfies $d_T = 2 < D < d = 3$. In figure 1 the expression $2R = 5.75 M^{1/2.51}$, which represents the functional dependence between size and mass averaged over all experiments, is plotted. This figure additionally exhibits for each value of mass the vertical bars associated with the experimental dispersion of diameter measurements. In figure 2 we give a diagram indicating the frequency distribution of the D obtained in the experiments.

The experimental dependence of D with the surface density σ was investigated in the interval $16\ \mathrm{g\ m^{-2}} \leqslant \sigma \leqslant 150\ \mathrm{g\ m^{-2}}$. We find that $D = c\sigma^{\alpha}$, with $c = 4.1$ and $\alpha = -\frac{1}{9}$. Then, for $\sigma \equiv \sigma_3 \sim 15\ \mathrm{g\ m^{-2}}$, $D = 3$ and for $\sigma \equiv \sigma_2 \sim 625\ \mathrm{g\ m^{-2}}$, $D = 2$. The error bars in this case have the average value $(\overline{\Delta D/D}) = 0.058$. We consider this expression involving D and σ as an effective relation not claimed to be valid in an asymptotic sense.

Further experiments have shown that $D \simeq \frac{5}{2}$ for crumpled paper balls formed from sheets of paper with area A in the interval $64\ \mathrm{mm^2} \leqslant A \leqslant 10^6\ \mathrm{mm^2}$ (corresponding to $1 \leqslant M \leqslant 2^{14}$) and $60\ \mathrm{g\ m^{-2}} \leqslant \sigma \leqslant 80\ \mathrm{g\ m^{-2}}$.

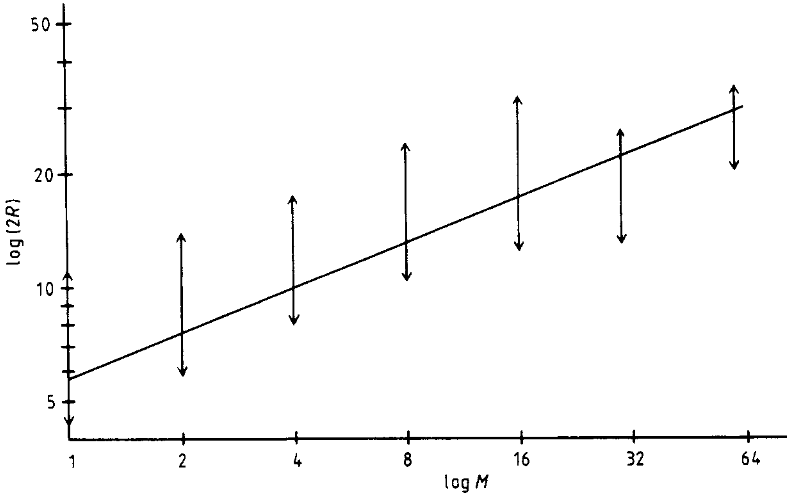

Figure 1. Log-log graph of the diameter ($2R$) against relative mass (M) for crumpled paper balls (see text). The surface density of the paper is $\sigma \simeq 80\ \mathrm{g\ m^{-2}}$.

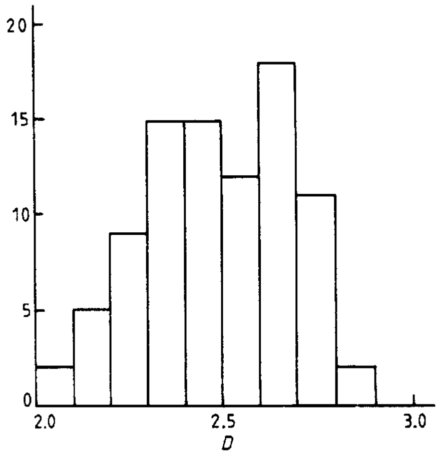

Figure 2. Frequency distribution of the fractal dimension D in crumpled paper balls (surface density of the paper $\sigma \simeq 80\ \mathrm{g\ m^{-2}}$) obtained from 89 experiments, $D = 2.51 \pm 0.19$.

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Note added. A referee pointed out to us a similar experiment mentioned by Kantor *et al* (1986).

References

Kantor Y, Kardar M and Nelson D R 1986 *Phys. Rev. Lett.* **57** 791-4

Mandelbrot B B 1983 *The Fractal Geometry of Nature* (San Francisco: Freeman)